

# An Adaptive Power and Bit Allocation Algorithm for MIMO OFDM/SDMA System Employing Zero-Forcing Multi-user Detection

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**Abstract**— The paper describes an adaptive algorithm for power and bit allocations in a multiple user Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing (MIMO OFDM) system with Space Division Multiple Access, which operates in a frequency selective fading channel. The zero forcing (ZF) technique is applied to accomplish multi user detection (MUD). A Lagrange multiplier method is applied to obtain a one-step solution for optimal power and bit allocations in this system. The resulting algorithm is advantageous over an alternative Greedy algorithm, because it does not require a time-consuming iterative procedure for its implementation. The algorithm assigns bits and power for all users according to the channel state information (CSI), which is assumed to be fully or partially available to the transmitter. The simulation results show the proposed algorithm operates successfully in multiple user access scenarios.

**Index Terms**—MIMO systems, OFDM, Lagrange multiplier method, Multiuser channel, Adaptive system.

## I. INTRODUCTION

RECENT years have seen increasing attempts to extend broadband multimedia services to mobile wireless networks. As the result of these efforts the MIMO OFDM system, being the combination of Multiple Input Multiple Output (MIMO) diversity techniques [1]-[2] and Orthogonal Frequency Division Multiplex (OFDM) technique [3], is envisaged as a candidate for realizing future wireless communications, such as described by the IEEE 802.16 standard [4].

In order to provide the desired quality of service (QoS) to users, an adaptive scheme with respect to power allocation [5], [6], modulation [7], beamforming [8], subcarrier allocation [9]-[10] or transmission data rate [11] can be applied. Practical implementations of such adaptive schemes are under way and they concern standards such as HIPERLAN/2 [12], UMTS-HSDPA [13]-[14], and GPRS-136 [7]. The key factor in practical implementation of these adaptive schemes is availability of a fast adaptive algorithm.

Many adaptive algorithms for MIMO OFDM system have already been proposed. However, only a few concern multiple user access scenarios [9]-[10], [15]-[17]. It has to be noted that most of them try to overpass the issue of multiple access interference (MAI), which is considered as co-channel interference (CCI). For example, in [10] and [15], the authors do not allow users to share the same subcarrier. As a result, there is no CCI in each subcarrier. However, this approach results in an inefficient use of the available frequency spectrum. In [16], the authors exploit an orthogonal method to solve the CCI problem. This method imposes some constraints on the number of used antennas and sacrifices the antenna diversity gain. In [17], redundant OFDM symbols are used to eliminate CCI.

In order to overcome the above shortfalls, a space division multiple access (SDMA) technique, as described in [18], can be used to allow users to share the same frequency band. This spatial multiplexing to separate users is of advantage in comparison with other techniques supporting multiple user access such as OFDMA [19], OFDM-CDMA [20] or MC-CDMA [21]. The reason is that these alternative techniques consume more bandwidth than the SDMA technique.

In this paper, we extend the work described in [9] and propose an adaptive power and bit allocation algorithm for a multi user MIMO OFDM/SDMA system. This algorithm is based on the Lagrange multiplier method and is performed using a one-step procedure. This algorithm is advantageous from the point of short execution time, as no iterations are required to obtain an optimal solution. In our investigations CCI is taken into account, as every user is allowed to share the same subcarrier with an arbitrary number of antennas at transmitter and receiver. This assumption allows for exploiting the system diversity in the frequency, space or user domain while maintaining the guarantee QoS for each user under the condition of the minimum transmitted power. The allocation procedure of bit and power is carried out according to the channel state information (CSI), which is assumed to be fully or partially available to the system transmitters. The solution to this problem is given in the close-form and its validity is verified via Monte Carlo simulations. The presented results show tradeoffs between QoS, number of users and minimum transmitted power for an assumed user capacity.

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The paper is organized as follows. In Section II, the channel model and the structure of transmitter and receiver of an adaptive MIMO OFDM/SDMA system are described. Section III introduces an optimization problem and provides its close-form solution. In Section IV, simulation results are shown and comparisons between the performance of the proposed adaptive system and its non adaptive counter part are presented. Section V concludes the paper.

## II. SYSTEM MODEL

### A. Transmitter of adaptive MIMO OFDM/SDMA system

The configuration of the transmitter of an adaptive MIMO OFDM/SDMA system is shown in Fig.1. This configuration is the same for all  $K$  users which includes  $N_T$  transmitting antennas and various data processing modules. As seen in Fig. 1, the tasks of adaptive modulation, power allocation and beamforming are divided into separate blocks. They require some information about the channel, which is obtained via a feedback loop from the receiver. It is assumed that the receiver knows the channel in a perfect way (for example from training sequences).

For the  $k$ th user, it is assumed that  $B_k$  bits correspond to one OFDM symbol. In this case,  $B_k$  can be interpreted as data transmission rate with  $B_k$  bits per one symbol period. In each symbol duration, a data stream composed of  $B_k$  bits is fed into  $N_C$  parallel streams, each containing  $b_{k,1}, b_{k,2}, \dots, b_{k,N_C}$  bits. These data streams are modulated into a symbol sequence  $s_{k,1}, s_{k,2}, \dots, s_{k,N_C}$  to be transmitted on  $N_C$  subcarriers. Each symbol  $s_{k,m}$  is scaled to a unit power and the transmitted power of each symbol is defined as  $\delta_{k,m}$  for the  $m$ th subcarrier. This information is required to make power adjustments over the subcarriers. In order to perform adaptive beamforming, the scaled symbols are multiplied with the basis beam vector  $\mathbf{v}_{k,m}$  being of  $N_T \times 1$  size. At the end of the transmitter, Inverse Fast Fourier Transform (IFFT) including cyclic prefix insertion ( $N_{CP}$  subcarriers) is performed. Values of  $b_{k,m}, s_{k,m}, \delta_{k,m}$  and  $\mathbf{v}_{k,m}$  are adjusted by an adaptive algorithm according to the feedback channel information.

### B. MIMO channel modeling

It is assumed that the investigated MIMO OFDM system operates in a frequency selective fading channel [8], [22] whose characteristics stay the same during transmission of one OFDM symbol. The fading channel between the  $i$ -th transmit antenna and the  $j$ -th receive antenna is modeled by a discrete time baseband equivalent  $L-1$  order finite impulse response (FIR) filter with filter taps  $g_{k,ij}(l)$ , where  $l = 0, 1, \dots, L-1$ . It is assumed that the  $L$  taps are independent zero mean complex Gaussian random variables with variance  $\sigma_g^2(l)$  and  $\sigma_h^2 = \sum \sigma_g^2(l)$ . The set of these variances defines the power delay profile (PDP). Here, PDP is assumed to be an exponentially decaying function with the root mean square delay spread defined by IEEE 802.11a standard [23]. The

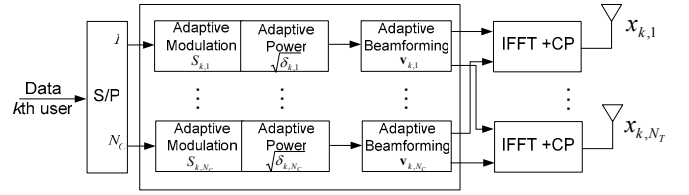


Fig. 1. Transmitter structure of adaptive MIMO OFDM/SDMA system

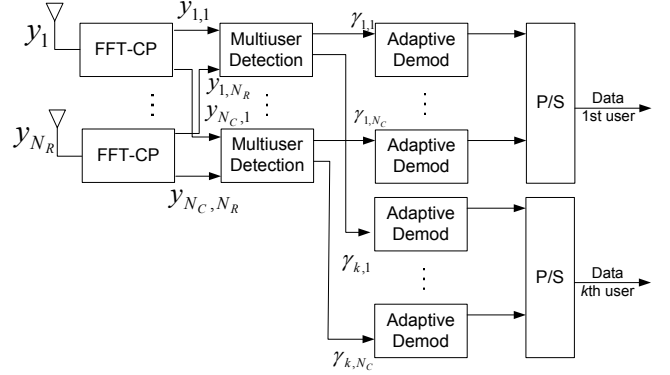


Fig. 2. Receiver structure of adaptive MIMO OFDM/SDMA system

discrete time MIMO baseband system at the  $t$ th time instant for  $k$ th user is described by the following equation

$$\mathbf{y}(t) = \sum_{k=1}^K \sum_{l=1}^L \mathbf{G}_k(l) \mathbf{x}_k(t-l) + \mathbf{n}(t) \quad (1)$$

where  $\mathbf{y}(t)$  is the complex  $N_R \times 1$  vector representing  $N_R$  received signals,  $\mathbf{G}_k(l)$  is the complex  $N_R \times N_T$  matrix consisting of the elements  $g_{k,ij}(l)$  and independent from  $k$ ,  $\mathbf{x}_k(t)$  is the complex  $N_T \times 1$  vector representing  $N_T$  transmitted signals and  $\mathbf{n}(t)$  is the complex  $N_R \times 1$  vector describing  $N_R$  additive and discrete-time noise at receiver.

Note that the inter symbol interference (ISI) can be eliminated by choosing  $L \leq N_C + N_{CP} + 1$ . As a result, the channel response between transmitter and receiver can be assumed to be flat on each subcarrier and defined by the complex  $N_R \times N_T$  matrices  $\mathbf{H}_{k,m}$ , where  $m$  denotes the subcarrier number.

### C. Receiver of adaptive MIMO OFDM/SDMA system

The configuration of the receiver of the investigated adaptive MIMO OFDM/SDMA system is shown in Fig. 2. It includes  $N_R$  antennas and data processing blocks whose functions are similar to those in the transmitter of Fig.1. Assuming perfect frequency synchronization, time synchronization and availability of tracking pilot subcarrier [24], the received signals of  $m$ th subcarrier is expressed as (2).

$$\mathbf{y}_m = \sum_{k=1}^K \mathbf{H}_{k,m} \mathbf{v}_{k,m} \sqrt{\delta_{k,m}} s_{k,m} + \mathbf{n}_m \quad (2)$$

where  $\mathbf{y}_m$  is the complex  $N_R \times 1$  vector representing a received signals  $y_{m,1}, y_{m,2}, \dots, y_{m,N_C}$  and  $\mathbf{n}_m$  is the complex  $N_R \times 1$  vector with variance  $N_0$ .

In the undertaken investigations, it is assumed that the CSI is perfectly known to the receiver. The transmitter obtains this information using feedback from the receiver. Two cases are considered, when perfect and partial CSI feedback between receiver and transmitter sites occurs.

### 1) Perfect CSI at Transmitter

The adaptive process is carried out according to the instantaneous CSI. For the system with time division duplex, this assumption is acceptable [9]. By applying the singular value decomposition (SVD) technique to channel matrix ( $\mathbf{H}_{k,m} = \mathbf{U}_{k,m} \mathbf{\Lambda}_{k,m} \mathbf{V}_{k,m}^\dagger$ ), the transmitter can select the basis beam vector  $\mathbf{v}_{k,m}$  along with the vector  $\mathbf{u}_{k,m}$  to achieve the maximum eigenvalue  $\lambda_{k,m}^{\max}$ . Note that  $(\cdot)^\dagger$  is the conjugate and transpose operation (Hermitian operation). Therefore, the received signal at  $m$ th subcarrier is given as

$$\mathbf{y}'_m = \mathbf{U}_m^\dagger \mathbf{U}_m \mathbf{\Lambda}_m \mathbf{s}_m + \mathbf{U}_m^\dagger \mathbf{n}_m \quad (3)$$

where  $\mathbf{U}_m$  is the  $N_R \times K$  matrix of  $[\mathbf{u}_{1,m}, \mathbf{u}_{2,m}, \dots, \mathbf{u}_{K,m}]$ ,  $\mathbf{\Lambda}_m$  is the  $K \times K$  diagonal matrix of  $\lambda_{k,m}^{\max}$  and  $\mathbf{s}_m$  is the  $K \times 1$  vector of  $\sqrt{\delta_{k,m}} s_{k,m}$ .

Here, Zero Forcing (ZF) technique is chosen for Multi User Detection (MUD). This is the most popular method because of ease of its implementation. By applying ZF technique, the decision can be expressed as

$$\mathbf{y}''_m = \mathbf{\Lambda}_m \mathbf{s}_m + (\mathbf{U}_m^\dagger \mathbf{U}_m)^{-1} \mathbf{U}_m^\dagger \mathbf{n}_m \quad (4.1)$$

$$y''_{k,m} = \lambda_{k,m}^{\max} \sqrt{\delta_{k,m}} s_{k,m} + \mathbf{d}_{k,m} \mathbf{U}_m^\dagger \mathbf{n}_m \quad (4.2)$$

where  $(\mathbf{U}_m^\dagger \mathbf{U}_m)^{-1} = [\mathbf{d}_{1,m}, \mathbf{d}_{2,m}, \dots, \mathbf{d}_{K,m}]^T$ ,  $(\cdot)^T$  is the transpose operation. To consider only the  $k$ th user in  $m$ th subcarrier, the hard decision can be achieved by  $y''_{k,m}$  and then the SNR can be written in (5).

$$\text{SNR}_{k,m} = \frac{(\lambda_{k,m}^{\max})^2 \delta_{k,m}}{c_{k,m} N_0} \quad (5)$$

where  $c_{k,m} = \|\mathbf{d}_{k,m} \mathbf{U}_m^\dagger\|_F^2$  is the spatial correlation between  $k$ th user and the other users. Note that  $\|\cdot\|_F$  denotes the Frobenius norm. This spatial correlation is related to the strength of CCI in the system. As observed in (5), this factor is independent from power allocation ( $\delta_{k,m}$ ), making the optimization problem more feasible to solve. For the single user case, the parameter  $c_{k,m}$  is equal to 1. The value of  $c_{k,m}$  is greater than 1 when the number of users increases. The SNR under multiple user environments is lower than for the case of single user. To maintain the same QoS under multiple user condition, more power has to be transmitted. This is confirmed by simulation results in the Section IV.

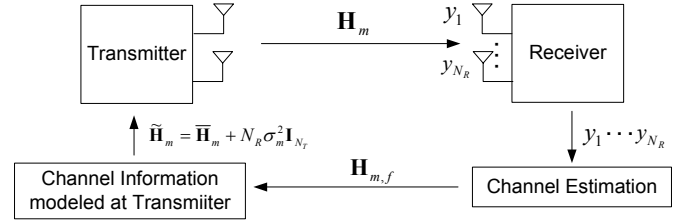


Fig. 3. Model of feed back channel information

### 2) Partial CSI at Transmitter

In practice, the perfect CSI at transmitter is difficult to achieve. Therefore the assumption of partial CSI is more meaningful from the practical point of view. Here, the concept of channel mean feedback, as introduced in [8], [22], is adopted for the partial CSI model. Fig. 3 illustrates the basic block diagram for feedback channel information used in this work. Using this concept, the channel information of  $m$ th subcarrier at the transmitter is modeled as  $\tilde{\mathbf{H}}_{k,m}$  with the mean value of  $\bar{\mathbf{H}}_{k,m}$  and the covariance matrix of  $N_T \tilde{\sigma}_{k,m}^2 \mathbf{I}_{N_R}$ .  $\bar{\mathbf{H}}_{k,m}$  is the conditional mean of  $\tilde{\mathbf{H}}_{k,m}$  when the feedback channel information  $\mathbf{H}_{k,m,f}$  is arrived at transmitter.

Therefore  $\tilde{\mathbf{H}}_{k,m}$  is a delayed version of  $\mathbf{H}_{k,m,f}$ . It is characterized by the correlation coefficient from Jakes' model:  $\rho_k = J_0(2\pi f_d \tau)$ , where  $J_0(\cdot)$  is the 0th order Bessel function,  $f_d$  is the maximum Doppler frequency and  $\tau$  is the feedback delay. When the channel information is fed back to the transmitter with the time delay  $\tau$  but without errors, the mean channel is given as  $\bar{\mathbf{H}}_{k,m} = \rho_k \mathbf{H}_{k,m,f}$  with the variance  $\tilde{\sigma}_{k,m}^2 = (1 - |\rho_k|^2) \sigma_h^2$ . Therefore at the transmitter the channel matrix  $\mathbf{H}_{k,m}$  can be modeled as  $\tilde{\mathbf{H}}_{k,m}$  as shown in (6).

$$E\{\tilde{\mathbf{H}}_{k,m}\} = \bar{\mathbf{H}}_{k,m} \quad (6.1)$$

$$E\{\tilde{\mathbf{H}}_{k,m} \tilde{\mathbf{H}}_{k,m}^\dagger\} = \bar{\mathbf{U}}_{k,m} (\bar{\mathbf{\Lambda}}_{k,m}^2 + N_T (1 - \rho_k^2) \sigma_h^2 \mathbf{I}_{N_R}) \bar{\mathbf{U}}_{k,m}^\dagger \quad (6.2)$$

By applying the above to (2) and selecting the basis beam vector  $\mathbf{v}_{k,m}$  along with the vector  $\bar{\mathbf{u}}_{k,m}$  to achieve the maximum eigenvalue  $\tilde{\lambda}_{k,m}^{\max}$ , one can rewrite the received signal in (3) as (7).

$$\mathbf{y}'_m = \bar{\mathbf{U}}_m^\dagger \mathbf{W}_m \tilde{\mathbf{\Lambda}}_m \mathbf{s}_m + \bar{\mathbf{U}}_m^\dagger \mathbf{n}_m \quad (7)$$

where  $\mathbf{W}_m$  is the  $N_R \times K$  matrix of  $\tilde{\mathbf{H}}_{k,m} \mathbf{v}_{k,m} / \tilde{\lambda}_{k,m}^{\max}$ .

After applying ZF technique, the solution can be expressed as given in (8)

$$y''_{k,m} = \tilde{\lambda}_{k,m}^{\max} \sqrt{\delta_{k,m}} s_{k,m} + \bar{\mathbf{d}}_{k,m} \bar{\mathbf{U}}_m^\dagger \mathbf{n}_m \quad (8)$$

where  $(\bar{\mathbf{U}}_m^\dagger \mathbf{W}_m)^{-1} = [\bar{\mathbf{d}}_{1,m}, \bar{\mathbf{d}}_{2,m}, \dots, \bar{\mathbf{d}}_{K,m}]^T$ . By

considering only the  $k$ th user in  $m$ th subcarrier, the hard decision can be achieved by  $y_{k,m}''$  and thus the SNR can be written in (9).

$$\text{SNR}_{k,m} = \frac{E\left\{\left|\tilde{\lambda}_{k,m}^{\max} \sqrt{\delta_{k,m}} s_{k,m}\right|^2\right\}}{E\left\{\left|\bar{\mathbf{d}}_{k,m} \bar{\mathbf{U}}_m^\dagger \mathbf{n}_m\right|^2\right\}} \quad (9)$$

$$= \frac{(\rho_k^2 (\lambda_{k,m}^{\max})^2 + N_T (1 - \rho_k^2) \sigma_h^2) \delta_{k,m}}{\bar{c}_{k,m} N_0}$$

where  $\bar{c}_{k,m} = E\left\{\left\|\bar{\mathbf{d}}_{k,m} \bar{\mathbf{U}}_m^\dagger\right\|_F^2\right\}$ . One can notice that the expression in (9) is the same as in (5) when  $\rho_k=1$ . This means that the signal to noise ratio for partial CSI case is equal to the one for the perfect case when the quality of feed back is best (without errors), or there is no delay in the feedback loop.

### 3) Bit error rate approximation

To simplify the task of evaluating bit error rate (BER), a unified expression of approximate BER [22] for QAM modulation is given in (10).

$$\text{BER}_{k,m} \approx 0.2 \exp(-g_{k,m}(b_{k,m}) \text{SNR}_{k,m}) \quad (10)$$

where the constant  $g_{k,m}(b_{k,m})$  depends on whether the chosen constellation is rectangular or square QAM:

$$g_{k,m}(b_{k,m}) = \begin{cases} \frac{6}{5 \cdot 2^{b_{k,m}} - 4} & b_{k,m} = 1, 3, 5, \dots \\ \frac{6}{4 \cdot 2^{b_{k,m}} - 4} & b_{k,m} = 2, 4, 6, \dots \end{cases}$$

## III. ADAPTIVE ALGORITHM

Before considering an adaptive technique for MIMO OFDM systems, first we comment on the works reported in [8]-[10]. In these works, the optimal subcarrier power and bit allocations were obtained using so-called Greedy algorithm. One has to note that this algorithm is of high computational complexity and yields only a one bit optimal solution. Computationally more efficient algorithms were proposed in [25]. However, they still require an iterative procedure for their implementation. This delays obtaining an optimal solution and affects QoS [26]. The shortcomings of the Greedy algorithm motivate the search for a new adaptive algorithm which would minimize a delay in signal processing.

Here, we adapt a one-step algorithm based on a Lagrange multiplier method, which was introduced in [27]-[28]. By considering a single user MIMO OFDM system, it has already been shown in [28] that the new algorithm for power and bit allocation is much faster than the Greedy algorithm. Here, we extend this algorithm to the multi user case.

By having a close look into expressions (5) (9) and (10), one can see that there are three main parameters apart from the knowledge of channel information, which directly influence

the system performance. These are BER ( $\text{BER}_{k,m}$ ), transmitted SNR ( $\delta_{k,m}/N_0$ ) and a number of loaded bits ( $b_{k,m}$ ). Here, the aim of optimization is to minimize the total transmitted power ( $P_T$ ) while keeping a constant data rate transmission ( $B_k$ ). The guaranteed BER is defined as the target Bit Error Rate ( $\text{BER}_{\text{Target}}$ ).

Using the above assumptions, the optimization problem is formulated as follows:

$$\arg \min_{b_{k,m}} \sum_{k=1}^K \sum_{m=1}^{N_C} \delta_{k,m} \quad (11.1)$$

$$\text{subject to: } \begin{cases} \sum_{m=1}^{N_C} b_{k,m} = B_k & \forall k \\ \delta_{k,m} > 0 \\ b_{k,m} > 0 \\ \text{BER}_{k,m} = \text{BER}_{\text{Target}} \end{cases} \quad (11.2)$$

For this algorithm, the optimization process is defined by equation (12)

$$\frac{\partial}{\partial b_{k,m}} \left( \sum_k \sum_m \delta_{k,m} + \sum_k \mu_k \left( \sum_m b_{k,m} - B_k \right) \right) = 0 \quad (12)$$

where  $\mu_k$ , being the constant value, is a Lagrange multiplier factor. Under the constraining conditions, the solution can be obtained using the assumption of perfect or partial CSI.

For the perfect CSI case, the solution is given in (13).

$$b_{k,m} = \frac{1}{N_C} \left\{ B_k + \log_2 \left( \frac{\left( (\lambda_{k,m}^{\max})^2 / c_{k,m} \right)^{N_C}}{\prod_{m'=1}^{N_C} (\lambda_{k,m'}^{\max})^2 / c_{k,m'}} \right) \right\} \quad (13)$$

For the partial CSI case, the solution is given in (14).

$$b_{k,m} = \frac{1}{N_C} \left\{ B_k + \log_2 \left( \frac{\left( (\rho_k^2 (\lambda_{k,m}^{\max})^2 + N_T (1 - \rho_k^2) \sigma_h^2) / \bar{c}_{k,m} \right)^{N_C}}{\prod_{m'=1}^{N_C} (\rho_k^2 (\lambda_{k,m'}^{\max})^2 + N_T (1 - \rho_k^2) \sigma_h^2) / \bar{c}_{k,m'}} \right) \right\} \quad (14)$$

From (13) and (14), it can be noticed that the optimal loaded bits for all subcarriers can be obtained by performing only one-step formula calculations. However, as  $b_{k,m}$  has to be a positive integer number, the following refinement is necessary:

- 1) Change  $b_{k,m}$  to be the nearest positive integer.
- 2) Find bit remaining (or overloading):  $B_{k,rem} = B_k - \sum b_{k,m}$
- 3) Add (or delete) one bit from  $B_{k,rem}$  subcarriers with respect to their maximum eigenvalue  $\lambda_{k,m}^{\max}$  in decreasing order.

Because the obtained solution does not require any iteration, the resulting algorithm is very fast. By assuming no limitation of computational processing power, it has already been shown in [28] that this algorithm is about  $B_k$  faster than

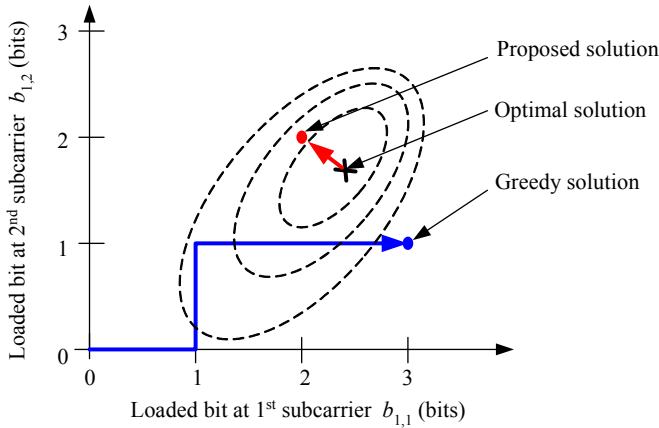


Fig. 4. Illustration of searching methods by Lagrange and Greedy algorithms.

the iterative Greedy algorithm. This time execution ratio indicates that the new algorithm can be of considerable advantage in high data rate transmission systems.

#### A. Search method for the optimal solution

Before assessing the performance of the adaptive system using the proposed algorithm, further explanations concerning the obtained solution are given. In order to simplify this task, only 2 subcarriers and a single user are considered with  $B_k = 4$ . Fig. 4 illustrates the search methods for the case of Greedy and Lagrange algorithms.

The optimal solution for two subcarriers ( $b_{1,1}$  and  $b_{1,2}$ ) is obtained using (14). This solution is the optimal solution marked in Fig. 4. The loaded bits for 1<sup>st</sup> and 2<sup>nd</sup> subcarrier are 2.4 and 1.6 bits. However, as the loaded bits of both subcarriers have to be positive integer numbers, therefore the solution from Lagrange algorithm is refined, as explained in the previous section. As seen in Fig. 4, the proposed solution is 2 bits for each subcarrier.

In turn, the Greedy algorithm tries to find the optimal solution bit by bit. As a result, it provides a different solution from the proposed algorithm, as seen in Fig. 4. Therefore, from these considerations it is hard to judge which algorithm is the best. This is due to the constraint of selecting positive integer numbers representing the valid solution.

#### B. Further comparisons with Greedy algorithm

Greedy algorithm is the most commonly used algorithm with respect to adaptive MIMO OFDM systems [8]-[10]. As already pointed out, the main advantage of the newly proposed (Lagrange) algorithm over the Greedy algorithm is its short processing time. It is approximately  $B_k$  times faster than the Greedy algorithm. However, this claim needs to be supported by similar performances, for example in terms of BER.

Therefore to complete the claim, the performance comparisons with the Greedy algorithm are carried out next. To obtain a fair assessment of benefits of the proposed adaptive algorithm with respect to bit and power allocations, a non adaptive system is also considered. It is assumed that the non adaptive system does not have any knowledge of CSI at

TABLE I  
Processing Time Comparison Between Greedy and Lagrange Algorithms  
(The unit is a millisecond per one OFDM symbol)

Algorithm	Minimize Transmitted Power		
	$B_k = 24$ bytes	$B_k = 240$ bytes	$B_k = 2400$ bytes
Greedy algorithm	26 ms	270 ms	2,837 ms
Lagrange method	10 ms	10 ms	10 ms

These values are evaluated by MATLAB 7.0 on a PC with Pentium(R) 4 CPU 2.4 GHz, 248 MB of RAM.

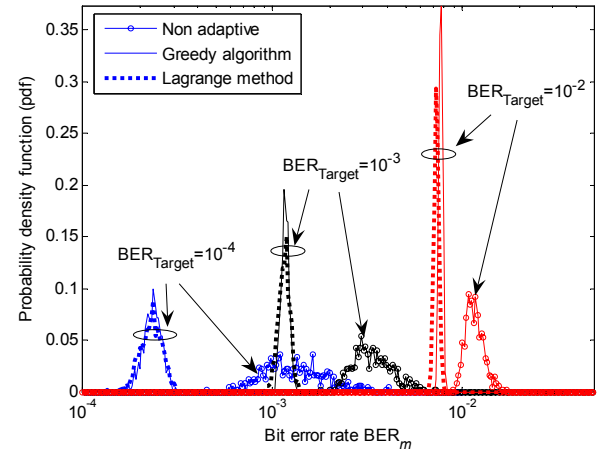


Fig. 5. Probability density function of bit error rate  $BER_m$  for  $K = 1$ ,  $\rho_k = 1$ ,  $N_C = 48$ ,  $B = 24$  bytes and  $N_R = N_T = 2$ .

transmitter and it allocates the same number of bits ( $B_k/N_C$ ) and amount of power ( $P_T/KN_C$ ) in each subcarrier. For this system, the weighting vector  $\mathbf{v}_{k,m}$  is given as the vector of all  $1/\sqrt{N_T}$ . The fully adaptive system has a full or partial knowledge of CSI at transmitter so it is able to allocate bits and power according to (13) or (14). Also it is able to adapt the vector  $\mathbf{v}_{k,m}$  (describing the beamforming process).

Table I shows the processing time of both algorithms for minimization of transmitted power. The results are evaluated within one OFDM symbol period for  $K = 1$ ,  $\rho_k = 1$ ,  $N_C = 48$ ,  $N_R = N_T = 2$  and  $BER_{\text{target}} = 10^{-3}$ . The total loaded bits  $B_k$  are varied by 24, 240 and 2400 bytes. The processing time of the Greedy algorithm increases as a function of  $B_k$ . As explained, the Lagrange algorithm requires the processing time, which is independent of  $B_k$ . For instance for  $B_k = 240$  bytes with 2,000 OFDM symbols sent, the Greedy algorithm is slower by 8.67 minutes of Pentium(R) 4 CPU 2.4 GHz, 248 MB of RAM.

Fig. 5 shows the probability density function (pdf) of bit error rate for  $K = 1$ ,  $\rho_k = 1$ ,  $N_C = 48$ ,  $B_k = 24$  bytes,  $N_R = N_T = 2$  and  $BER_{\text{target}} = 10^{-2}$ ,  $10^{-3}$ ,  $10^{-4}$ . The results were obtained from over 1,000 channel realizations. As observed in Fig. 5, the adaptive systems provide lower BER than the non adaptive one. For example for  $BER_{\text{target}} = 10^{-3}$ , the maximum BER probability of Greedy and Lagrange algorithms occur at  $1.1 \times 10^{-3}$  while for the non adaptive one it is  $3.5 \times 10^{-3}$ . Also the non-adaptive system offers larger variance (1.04) than the Greedy (0.25) and Lagrange (0.30) algorithms. This result shows that the two adaptive algorithms lead to more reliable

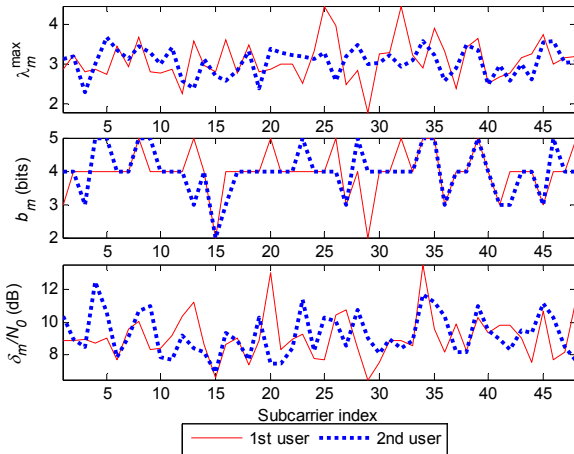


Fig. 6. The characteristic of maximum eigenvalue  $\lambda_{k,m}^{\max}$ , bit allocation  $b_{k,m}$  and power allocation  $\delta_{k,m}/N_0$  versus subcarrier index for multiple users,  $K=2$ ,  $N_C=48$ ,  $N_R=N_T=4$ .

system than the non-adaptive counter part. Also observed in Fig. 5 is the fact that when the target of bit error rate is changed, the BER probabilities of the two systems are adapted and are close to the target. These results indicate that the adaptive processes work well for both the Greedy and Lagrange algorithms.

By comparing all of the results in Table I and Fig. 5, it can be concluded that the two algorithms provide almost identical results with respect to the two main parameters of adaptive system ( $\text{BER}_{\text{target}}$  and minimum transmitted power). However, the algorithm based on the Lagrange multiplier method is advantageous in terms of processing time.

The next section concerns the numerical results produced by the proposed algorithm, which is based on the Lagrange method. The results from Greedy algorithm show the same trends, and therefore are not presented.

#### IV. SIMULATION RESULTS AND DISCUSSION

In this section, the performance of the proposed adaptive MIMO OFDM/SDMA system is investigated by Monte Carlo simulations in MATLAB. In simulations one OFDM symbol with 64 subcarriers and  $N_C=48$  data subcarriers [23] is assumed. The random 2,000 OFDM symbols are sent to evaluate BER performance and the results are averaged over 1,000 channel realizations. The PDP of the frequency selective fading channel is modeled by an exponential function with rms delay spread of 250 ns with 32 taps ( $L$ ). All channels are assumed to be Rayleigh fading channels. The results are simulated with the fixed bit rate transmission of 24 bytes ( $B_k=192$  bits) per one OFDM symbol. Simulations for other cases of bit rate transmission ( $B_k=1920$  and 19200 bits), number of antennas (2x2 and 8x8 MIMO systems), and number of subcarriers (32 and 128 subcarriers) were also carried out. However as they showed similar trends, they are not presented here.

##### A. Characteristic of Bit and Power allocations

Fig.6 shows the behavior of maximum eigenvalue  $\lambda_{k,m}^{\max}$ , bit

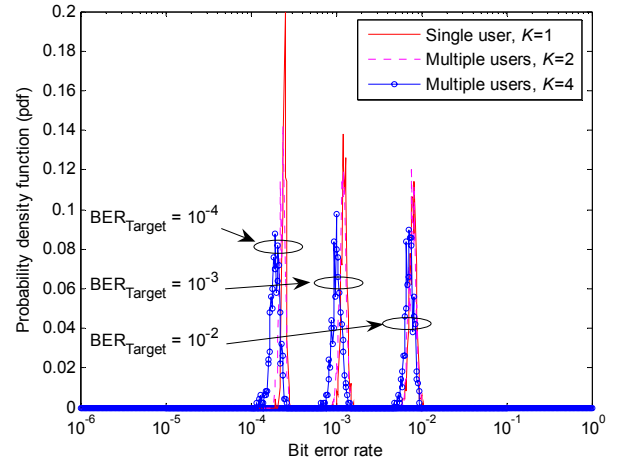


Fig. 7. Probability density function (pdf) of bit error rate for  $N_C=48$ ,  $\rho_k=1$  (Perfect CSI),  $B_k=24$  bytes and  $N_R=N_T=4$ .

allocation  $b_{k,m}$  and power allocation  $\delta_{k,m}/N_0$  of a 4x4 MIMO system over subcarrier frequency when two users are present in the system. The eigenvalue is evaluated at the transmitter during the adaptive process of bit allocations. It can be seen in Fig.6 that the maximum eigenvalue depends on an individual user. Loaded bits are allocated according to the condition of the minimum power in the system. It can be observed that the algorithm attempts to find an optimal solution for each user individually. Note that the total number of loaded bits for the users varies according to  $B_k$ . In the non-adaptive system and the adaptive only beamforming system each subcarrier is equally allocated with 4 bits leading to a fixed 16 QAM modulation.

##### B. Performance of adaptive algorithm

Fig. 7 shows the probability density function (pdf) of bit error rate when CSI is fully available to the transmitter ( $\rho=1$ ). The remaining parameters are chosen as  $N_C=48$ ,  $B_k=24$  bytes,  $N_R=N_T=4$  and  $\text{BER}_{\text{target}}=10^{-2}, 10^{-3}, 10^{-4}$ . One, two and four users are assumed to be present in the system. As observed in Fig. 7, the pdf distributions are similar for the varying numbers of users and are clustered around the target BER. These results reveal that the proposed adaptive process for multiple users operates successfully, as power and bits are correctly allocated to achieve the desired BER.

Table II shows the results for the average energy bit per noise ratio  $E_b/N_0$  (dB) for the same system. This power is the minimum power required for keeping multiple users to achieve the same target BER. As observed in the presented results, when four users are present in the system, each of them requires  $E_b/N_0$  to be at least 6.95dB and 9.87dB to maintain BER at receiving end of  $10^{-2}$  and  $10^{-3}$  respectively. In turn, a single user requires only -1.20dB and 2.11dB for the respective two cases of BER. As seen in Table II, the minimum power increases when the number of users increases. Also, the minimum power increases when an improvement of quality of service (smaller value of BER) is demanded. This can be explained by the fact that the spatial

TABLE II  
TRANSMITTED POWER ALLOCATED BY ADAPTIVE ALGORITHM

BER <sub>Target</sub>	Average energy bit per noise ratio $E_b/N_0$ (dB)		
	1 user	2 users	4 users
$10^{-2}$	-1.20	0.87	6.95
$10^{-3}$	2.11	3.39	9.87
$10^{-4}$	3.71	4.93	12.78

correlation ( $c_{k,m}$ ) increases when the number of users is increased. Their presence degrades the overall SNR performance in the system. As a result, BER<sub>Target</sub> is required to be lower when the higher SNR is needed.

Table II provides a useful summary, which can be of practical value when planning the user capacity in the system. For instance, when considering the base station design which always has the limited transmitted power; one has to make a compromise between QoS and a number of users, as shown in Table II.

### C. BER performance of adaptive and non adaptive systems

Fig. 8 presents the BER performance for a different number of users. It is clearly seen that no matter what number of users is assumed, the adaptive system outperforms the non adaptive system. For instance, when  $K = 2$ , the adaptive system requires only 3 dB for  $E_b/N_0$  to achieve BER at  $10^{-3}$  while the non adaptive system has needs 11 dB. This trend is valid for the other presented numbers of users. The energy bit used in the adaptive system for two users is much lower than for a single user in the non adaptive system. It means that for a small number of users, the adaptive system shows benefits in terms of BER and user capacity.

### D. BER performance of perfect and partial CSI

Fig. 9 shows the effect of feedback quality in the fully adaptive system on the bit error rate performance versus average energy bit per noise ratio  $E_b/N_0$  (dB) for multiple users,  $K = 2$ ,  $N_C = 48$ ,  $B_k = 24$  bytes and  $N_R = N_T = 4$ . The results show that the BER performance declines with poorer feedback of CSI between the receiver and the transmitter. However, even when  $\rho_k = 0.5$ , the performance of the fully adaptive system is by far superior over the non adaptive system.

## V. CONCLUSION

In this paper an adaptive algorithm for bit and power allocations in a multiple user MIMO OFDM/SDMA system operating in a frequency selective fading channel has been described. The close-form solution for allocating loaded bits according to the assumed target BER has been presented. The proposed algorithm is based on the Lagrange multiplier method and involves only a one step procedure, which leads to its very fast execution time. It has been shown that without compromising the system performance this algorithm is much faster than the Greedy algorithm, which is frequently used in conjunction with MIMO OFDM systems. The performances

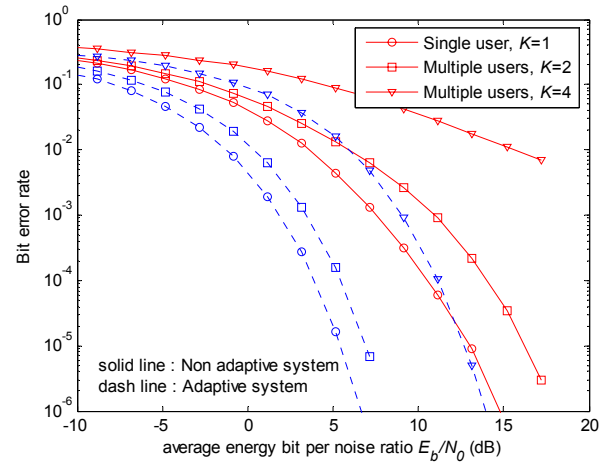


Fig. 8. Bit error rate vs. average energy bit per noise ratio  $E_b/N_0$  (dB) for single user,  $K = 1$ ,  $N_C = 48$ ,  $\rho_k = 1$ ,  $B_k = 24$  bytes and  $N_R = N_T = 4$ .

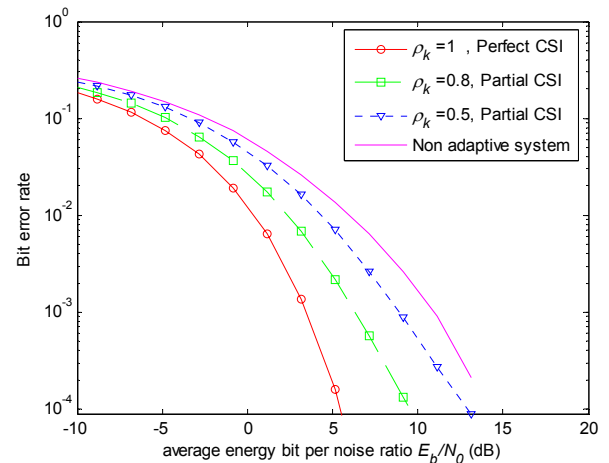


Fig. 9. Bit error rate vs. average energy bit per noise ratio  $E_b/N_0$  (dB) for multiple users,  $K = 2$ ,  $N_C = 48$ ,  $B_k = 24$  bytes and  $N_R = N_T = 4$ .

of the adaptive and non adaptive MIMO OFDM systems have been compared via Monte Carlo simulations. It has been shown that the use of the proposed adaptive algorithm leads to superior performance of the MIMO OFDM/SDMA system. Because of its short execution time, this algorithm should be of considerable interest to the designers of adaptive MIMO OFDM/SDMA systems.

## ACKNOWLEDGMENT

The authors acknowledge the financial support of the Australian Research Council via Grant **DP0450118**.

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